Lecture 3: Head-To-Head Comparisons, Condorcet Winner.

If we know the preference rankings of the voters, we can compare the votes of any two candidates to see which one would win in a plurality election in the absence of the other candidates. Such a comparison is called a **Head-To-Head** comparison of the two candidates.

Example There are 3 candidates for the position of President for the Notre Dame Squash Club. The preferences of the five members of the selection committee are shown in the following table:

Presidential preference rankings

#Voters	**1**	**2**	**1**	**1**
Colley	1	2	2	3
Henry	2	1	3	2
Taylor	3	3	1	1

(a) Which candidate would win in a head-to-head comparison between Colley and Henry?

(b) Which candidate would win in a head-to-head comparison between Colley and Taylor?

(c) Which candidate would win in a head-to-head comparison between Henry and Taylor?

In the above example, Henry beat all of the other candidates in a head-to-head comparison. Such a candidate is an example of a Condorcet winner, named after the Marquis de Condorcet (1743-1794).

Condorcet Winner A candidate who is the winner of a head-to-head comparison with every other candidate is called a **Condorcet winner**. If a candidate beats or ties with every other candidate in a head-to-head comparison, that candidate is called a **weak Condorcet winner**. For any election, there may or may not be a Condorcet winner or a weak Condorcet winner.

Example In the example above, Henry is a Condorcet winner.

We can **shorten our calculations** to find a Condorcet winner. We can begin with two strong candidates and compare them. If one wins, then we can eliminate the loser of that comparison from future calculations. We compare the winner to a third candidate, eliminate the loser from our calculations and compare the winner to a fourth candidate, etc, etc, etc..... This reduces the number of calculations significantly.

Example Is there a Condorcet winner in our video contest?

# Voters	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**2**	**1**
Blank Space by Taylor Swift	1	5	3	3	2	3	1	2	4
I won't let you down by OK Go	3	1	2	2	1	2	4	1	2
Never Catch me by Flying Lotus	4	3	1	4	5	5	2	3	1
Night Changes by One Direction	2	4	4	1	3	1	3	4	5
Water Fountain by tUnE yArDs	5	2	5	5	4	4	5	5	3

Example Check that there is no Condorcet winner in the election for which the preferences of the voters are shown below.

#Voters	**1**	**1**	**1**
Alpha	1	3	2
Beta	2	1	3
Gamma	3	2	1

In the following example, we see that a Condorcet winner may not even make it to the runoff in a plurality election with a runoff.

Example In a poll to determine player of the year for the North Bend Tennis Club, the ranking preferences of the 100 voters are shown below.

#Voters	**9**	**28**	**11**	**23**	**23**	**6**	**R1**	**R2**
Federer	1	1	2	3	2	3		
Williams	2	3	1	1	3	2		
Nadal	3	2	3	2	1	1		

(a) Calculate the winner of an Instant Runoff election.

(b) Is there a Condorcet Winner?

In the following example, we see that a Condorcet winner may not be the winner in a Borda count (average rankings).

Example (Purely Fictitious) The following teams participated in 11 events in the Celtic Games 2009: Ireland, Wales, Scotland, Brittany and Cornwall. The rankings of the teams in the events and the number of events in which each given ranking occurred are shown in the table below. It remains for the tournament organizers to determine which team should be declared the winner of the tournament (Who should be Number 1?)

#Events	**2**	**1**	**2**	**1**	**1**	**4**
Brittany	1	3	5	5	5	5
Cornwall	2	1	1	3	3	4
Ireland	3	2	2	1	2	3
Scotland	4	4	3	2	1	1
Wales	5	5	4	4	4	2

(a) Who wins with the Borda method?

(b) Who wins a Plurality of the vote?

(c) Who wins using the plurality method with a runoff between first and second place winners?

(d) Which option, if any, is the Condorcet winner?

Notes

1. We have seen above that the Condorcet winner may not win under the Borda Count method and may not even make the runoff if the plurality with runoff method is used. Many argue that if a Condorcet winner exists, then that candidate should be declared the winner. (Think of our round robin tournament).

2. For three candidates, with voters voting randomly, the probability that there is a Condorcet winner is always above 90%. As the number of candidates (and the number of voters) increases , the probability of a Condorcet winner decreases; for six candidates it is always above 68%. In reality, there are often strong and weak candidates and voting is not random, this increases the liklihood of a Condorcet winner.

Condorcet Completion Process

In a contest where the Condorcet winner, if there is one, emerges victorious, a back-up procedure is necessary for a situation where no Condorcet winner emerges. A number of methods are available.

Black 1963 If no Condorcet winner exists, use Borda's method to decide the election.

Nanson
Round 1. : Use Borda count to eliminate all candidates with below average Borda count (= above average average rank).
Round 2. : Modify the original ranking of the voters to create new rankings for the remaining candidates. Calculate new Borda Counts (averages) and eliminate those with below average Borda count(above average average rank).
Round 3: Continue in this way until there is one candidate left.
Note if there is a Condorcet winner, that candidate will also be the winner using Nanson's method.

Copeland's method This is a simple method often used to decide a Round Robin Tournament, where every candidate plays every other candidate exactly once. With Copeland's method, candidates are ordered by the number of pairwise victories minus the number of pairwise defeats. This method is easily understood and easy to calculate, however it often leads to ties and puts more emphasis on the number of victories and defeats rather than their magnitude.

Method of Pairwise Comparisons (Equivalent to Copelands method) As with Copeland's method, candidates are compared in pairs. The one who wins gets one point for a win from such a comparison, 1/2 point for a tie and 0 points for a loss. The points from each pairwise comparison are added and the winner is the candidate with the most points.

Example Use Copeland's method and Nanson's method to decide the winner of the Nation's Cup in the Celtic games 2010, the result of which are shown below (there were only ten events in the 2010 games),

#Events	**3**	**3**	**2**	**1**	**1**	Ave.
Brittany	5	1	3	5	3	3.2
Cornwall	3	4	1	4	1	2.8
Ireland	1	2	4	2	5	2.4
Scotland	2	3	5	3	4	3.2
Wales	4	5	2	1	2	3.4
					Ave.	3

Copeland's Method In the following table, we will record a 1 if the row player wins in a head-to-head comparison between the row and column player, we will record a -1 if the column player wins and a 0 if the caparison is a draw. The sum of each row will give us W-L for the row player.

#Teams	Brittany	**Cornwall**	**Ireland**	**Scotland**	**Wales**	**W − L**
Brittany	−					
Cornwall		−				
Ireland			−			
Scotland				−		
Wales					−	